24.4 BONDS

Companies and corporations need money to run their business operations. Also, municipalities and governments require lot of money for infrastructure development and to run various welfare schemes for general public. To raise money these origanisations issue bonds for investors willing to lend money for a certain period of time. Thus, bonds are loans made to large organisations like municipalities, national governments and companies. In finance, a bond is an instrument of indebtness of the bond issuer to the bond holders. We may also describe the term 'bond' as follows.

BOND *A bond is a debt instrument that provides a periodic stream of interest payments to investors (or buyers) while repaying the principal sum on specific maturity date.*

A bond's terms and conditions are contained in legal contract between the buyer and the issuer, known as indenture.

The issuer of bonds agree to pay back the bond holder the face value of the bond on a specific date (normally called maturity date) and periodic stream of interest payments.

A bond is characterised by the following terms:

MATURITY *A bond's maturity is the time period until the principal is scheduled to be repaid.*

FACE VALUE *The face value (also known as par value) of a bond is the price at which the bond is sold to investors (or buyers) at the time of issue.*

It is also the price at which the bond is redeemed at maturity.

REDEMPTION PRICE *The redemption price of a bond is the amount the bond issuer pays at maturity.*

The redemption price is usually equal to the face value, in which case the bond is said to be redeemed at par. Sometimes, the bond may be redeemed at a price higher than the face value, in such cases, the bond is said to be redeemed above par. The redemption price is stated on the bond.

The statement "a bond is to be redeemed at 110" means that the redemption price of the bond is 110% of its face value.

DIVIDEND RATE *The rate at which a bond yields interest is called the dividend rate or the nominal interest rate.*

The dividend is always computed on the face value of the bond.

The issuer pays the interest at pre-determined intervals (usually semi-annually or annually) and returns principal on the maturity date.

COUPON RATE *A bond's coupon rate denotes the annual interest rate paid by the bond issuer to the bond holder.*

Coupon rate is determined as a percentage of the bond's face value or par value.

Bonds are bought in the open market at a price usually different from the redemption price or face value. This is due to the fact that the purchaser or investor expects to realize an income from his investment at a rate which is usually higher than the dividend rate. The price at which an investor purchases a bond is called the purchase price. The purchase price may be higher or lower than the face value of the bond. The return an investor realizes on a bond is called the bond yield. Note that bond yield is the annual net profit that an investor earns on an investment whereas dividend is the total interest earned on investment in a bond.

The owner of a bond is entitled to receive all future dividends and the redemption price at maturity.

FAIR PRICE *The fair price of a bond is the open market value that is acceptable to both the buyer and the seller.*

24.5 VALUATION OF BONDS

Bond valuation is a technique for determining the theoretical fair value or fair price of a particular bond. The fundamental principle of bond valuation is that its fair value is equal to the sum of present values of its expected cash flows. Cash flow is the cash that is estimated to be received in future from investment in a bond. There are only two types of cash flows that can be received from investments in bonds (i) coupon payments i.e. periodic interest payments (ii) principal payment at maturity i.e. payment received on redemption. The coupon payments are received at regular intervals as per the bond agreement, and final coupon payment plus principal payment is received at the maturity. There are different approaches to determine the fair value or fair price or purchase price of bonds. In this book, we will study the two main approaches viz. (i) Present value approach (ii) Relative price approach.

24.5.1 PRESENT VALUE APPROACH

In the present value approach, we first calculate the present value of each expected cash flow and then we add all the individual present values to obtain the value or fair value of purchase price of a bond. Let there be a bond with the

Face value = F

Redemption price or, Maturity value = C

Yield rate or market interest rate per period = i

Number of periodic payments or, Number of cash flows = n

Periodic interest (dividend) payment or, Coupon payment = R

Value of bond or Purchase price of bond or, Market price of bond = V

Then,

Bond value = Present value of first periodic payment + Present value of second periodic payment +...+ Present value of nth periodic payment + Present value of Maturity value or, Redemption price

$$\Rightarrow \quad \text{Bond value} = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \ldots + \frac{R}{(1+i)^n} + \frac{C}{(1+i)^n}$$

$$\Rightarrow \quad \text{Bond value} = \frac{R}{1+i} \cdot \frac{\left\{1 - \left(\frac{1}{1+i}\right)^n\right\}}{\left(1 - \frac{1}{1+i}\right)} + \frac{C}{(1+i)^n}$$

$$\Rightarrow \quad \text{Bond value} = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

REMARK *If a bond is redeemed at part, then $C = F$*

$$\therefore \quad \text{Bond value} = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + F\,(1+i)^{-n}.$$

24.5.2 RELATIVE PRICE APPROACH

In relative price approach, the bond is priced relative to yield to maturity (YTM) of a benchmark, usually a government bond of similar maturity or duration. This required return is then used to discount the cash flows, by replacing i in the present value approach, to obtain the market price of the bond.

ILLUSTRATIVE EXAMPLES

EXAMPLE 1 *A company ABC Ltd has issued a bond having a face value of ₹ 10,000 paying annual dividends at 8.5%. The bond will be redeemed at par at the end of 10 years. Find the purchase price of this bond if the investor wishes a yield rate of 8% (Given* $(1.08)^{-10} = 0.46319349$).

SOLUTION It is given that :

F = Face value of the bond = ₹ 10,000

n = Number of periodic dividend payments = Period of redemption = 10

i = Annual yield rate $= \frac{8}{100} = 0.08$

R = Annual dividend = 8.5 % of face value = $₹\left(\frac{8.5}{100} \times 10{,}000\right) = ₹\ 850$

The bond is redeemed at par.

$\therefore$ C = Redemption price or Maturity value = Face value = ₹ 10,000.

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[850\left\{\frac{1-(1+0.08)^{-10}}{0.08}\right\}+10{,}000\,(1+0.08)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\left[\frac{85000}{8}\{1-(1.08)^{-10}\}+10{,}000\,(1+1.08)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\,\{10{,}625\,(1-0.46319349)+10{,}000\times 0.46319349\}$$

$$\Rightarrow \quad V = ₹\,(10{,}625\times 0.53680651+10{,}000\times 0.46319349)$$

$$\Rightarrow \quad V = ₹\,(5703.57+4631.93) = ₹\,10{,}335.50$$

Hence, the purchase price of the bond is ₹ 10,335.50.

EXAMPLE 2 *A company has issued a bond having face value of* ₹ 100,000, *carrying an annual divided rate of 7% and maturing in 15 years. If the prevailing market rate of interest is 9%, and the bond is redeemed at par, find the bond value. Given* $(1.09)^{-15} = 0.27453804$).

SOLUTION We have,

F = Face value of the bond = ₹ 100,000

n = Period of redemption = Number of annual dividends = 15

i = Annual yield rate = $\frac{9}{100} = 0.09$

R = Annual dividend = 7 % of face value = ₹ $\frac{7}{100}\times 100{,}000 = ₹\,7{,}000$

The bond is redeemed at par.

$\therefore$ C = Redemption price or Maturity value = Face value = ₹ 100,000.

Let V be the bond value. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\}+C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[7{,}000\left\{\frac{1-(1+0.09)^{-15}}{0.09}\right\}+100{,}000\,(1+0.09)^{-15}\right]$$

$$\Rightarrow \quad V = ₹\left[\frac{700{,}000}{9}\{1-(1.09)^{-15}\}+100{,}000\,(1.09)^{-15}\right]$$

$$\Rightarrow \quad V = ₹\left\{\frac{700{,}000}{9}(1-0.27453804)+100{,}000\times 0.27453804\right\}$$

$$\Rightarrow \quad V = ₹\left(\frac{700{,}000}{9}\times 0.72546196+27453.804\right) = ₹\,(56424.819+27453.804) = ₹\,83878.62$$

Hence, the bond value is ₹ 83,878.62.

EXAMPLE 3 *A company has issued a bond having the face value of* ₹ 100,000 *carrying a coupon rate of 8% to be paid semi-annually and maturing in 5 years. If the prevailing market rate of interest is 7%, find the bond value. Given* $(1.035)^{-10} = 0.70891881$).

SOLUTION We have,

F = Face value of the bond = ₹ 100,000, n = Number of periods = $2\times 5 = 10$

i = Semi-annual yield rate = $\frac{7}{200} = 0.035$

R = Semi-annual dividend = ₹$\left(100{,}000\times\frac{8}{200}\right) = ₹\,4{,}000$

The bond is redeemed at par. Therefore,

$\therefore$ C = Redemption price or maturity value = Face value = ₹ 100,000.

Let V be the bond value. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[4{,}000\left\{\frac{1-(1+0.035)^{-10}}{0.035}\right\} + 100{,}000\,(1+0.035)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\left[\frac{4{,}000{,}000}{35}\{1-(1.035)^{-10}\} + 100{,}000\,(1.035)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\left\{\frac{4{,}000{,}000}{35}(1-0.70891881) + 100{,}000\times 0.70891881\right\}$$

$$\Rightarrow \quad V = ₹\left(\frac{4{,}000{,}000}{35}\times 0.29108119 + 70891.881\right) = ₹\,(33266.421 + 70891.881) = ₹\,104158.30$$

Hence, the bond value is ₹ 104158.30.

EXAMPLE 4 *A bond has a face value of* ₹ 10,000 *and matures in 15 years at part. The nominal interest is 7%. What is the price of the bond that will yield an effective interest of 8%? (Given* $(1.08)^{-15} = 0.31524170$*).*

SOLUTION We have,

F = Face value = ₹ 10,000, n = Number of periods = 15

i = Annual yield rate $= \frac{8}{100} = 0.08$, R = Annual dividend = ₹ $\frac{7}{100}\times 10{,}000$ = ₹ 700

C = Maturity value = Face value = ₹ 10,000.

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[700\left\{\frac{1-(1+0.08)^{-15}}{0.08}\right\} + 10{,}000\,(1+0.08)^{-15}\right]$$

$$\Rightarrow \quad V = ₹\left[\frac{70{,}000}{8}\left\{1-(1.08)^{-15}\right\} + 10{,}000\,(1.08)^{-15}\right]$$

$$\Rightarrow \quad V = ₹\left[\frac{70{,}000}{8}(1-0.31524170) + 10{,}000\times 0.31524170\right]$$

$$\Rightarrow \quad V = ₹\left(\frac{70{,}000}{8}\times 0.6847583 + 3152.4170\right) = ₹\,(5991.6351 + 3152.4170) = ₹\,9144.05$$

Hence, the purchase price of the bond is ₹ 9144.05.

EXAMPLE 5 *Find the purchase price of a* ₹ 20,000 *bond, redeemable at the end of 10 years at 110, and paying annual dividends at 4%, if the yield rate is to be 5% effective. (Given* $(1.05)^{-10} = 0.61391325$*).*

SOLUTION It is given that

n = Number of period = 10, i = Yield rate $= \frac{5}{100} = 0.05$

R = Annual dividend = 4% of face value = ₹ $\left(\frac{4}{100}\times 20{,}000\right)$ = ₹ 800

The bond is redeemed at 110. Therefore, the redemption price of the bond is 110% of its face value. Thus,

$$C = \text{Redemption price} = ₹\left(20{,}000 \times \frac{110}{100}\right) = ₹\ 22{,}000$$

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[800\left\{\frac{1-(1+0.05)^{-10}}{0.05}\right\} + 22{,}000\,(1+0.05)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\left[16{,}000\,\{1-(1.05)^{-10}\} + 20{,}000\,(1.05)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\,\{16{,}000\,(1-0.61391325) + 22{,}000 \times 0.61391325\}$$

$$\Rightarrow \quad V = ₹\,(16{,}000 \times 0.38608675 + 22{,}000 \times 0.61391325)$$

$$\Rightarrow \quad V = ₹\,(6177.388 + 13506.0915) = ₹\ 19683.48$$

EXAMPLE 6 *Find the purchase price of a ₹ 50,000, 6% bond, dividends payable semi-annually, redeemable at par in 10 years, if the yield rate is to be 5% compounded semi-annually. (Given* $(1.025)^{-20} = 0.61027094$*).*

SOLUTION It is given that

F = Face value of bond = ₹ 50,000, i = Yield rate $= \frac{5}{200} = 0.025$

R = Semi-annual dividend $= ₹\left(\frac{6}{200} \times 50{,}000\right) = ₹\ 1500$

n = Number of periodic payments $= 2 \times 10 = 20$

The bond is redeemable at par. Therefore, C = Redemption price = ₹ 50,000

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[1500\left\{\frac{1-(1+0.025)^{-20}}{0.025}\right\} + 50{,}000\,(1+0.025)^{-20}\right]$$

$$\Rightarrow \quad V = ₹\left[60{,}000\left\{1-(1.025)^{-20}\right\} + 50{,}000\,(1.025)^{-20}\right]$$

$$\Rightarrow \quad V = ₹\,[60{,}000\ (1-0.61027094) + 50{,}000 \times 0.61027094]$$

$$\Rightarrow \quad V = ₹\,(60{,}000 \times 0.38972906 + 50{,}000 \times 0.61027094) = ₹\,(23383.7436 + 30513.547) = ₹\ 53897.29$$

EXAMPLE 7 *Find the purchase price of ₹ 10,000 bond, redeemable at th end of 10 years at ₹ 11,000 and paying annual dividends at 4%, if the yield rate is to be 5% per annum effective. (Given* $(1.05)^{-10} = 0.61391325$*)*

SOLUTION We have,

F = Face value of the bond = ₹ 10,000

C = Redeemable value of the bond = ₹ 11,000

R = Annual dividend = ₹$\left(\frac{4}{100}\times 10{,}000\right)$ = ₹ 400

n = Number of periodic payments (dividends) = 10

i = Yield rate = $\frac{5}{100}$ = 0.05

Let V be the purchase price of the bond. Then,

$$V = R\left\{\frac{1-(1+i)^{-n}}{i}\right\} + C\,(1+i)^{-n}$$

$$\Rightarrow \quad V = ₹\left[400\left\{\frac{1-(1+0.05)^{-10}}{0.05}\right\} + 11{,}000\,(1+0.05)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\left[8{,}000\left\{1-(1.05)^{-10}\right\} + 11{,}000\,(1.05)^{-10}\right]$$

$$\Rightarrow \quad V = ₹\,[8{,}000\,(1-0.61391325) + 11{,}000\times 0.61391325]$$

$$\Rightarrow \quad V = ₹\,(8{,}000\times 0.38608675 + 11{,}000\times 0.61391325)$$

$$\Rightarrow \quad V = ₹\,(3088.694 + 6753.04575) = ₹\ 9841.73975 = ₹\ 9841.74$$

EXERCISE 24.3

1. A ₹ 100,000 bond paying annual dividends at 3.5% will be redeemed at par at the end of 6 years. Find the purchase price of this bond, if the investor wishes a yield rate of 4.5% (Given $(1.045)^{-6} = 0.76789574$).
2. A bond has face value of ₹ 10,000 and maturity period of 10 years. The nominal interest rate is 6%. What should be the price of the bond to yield an effective interest rate of 8%? (Given $(1.08)^{-10} = 0.46319349$).
3. An investor intends purchasing a 12 years ₹ 1,000 face value bond having nominal interest rate of 6%. At what price the bond may be purchased now if it matures at par and the investor requires a yield rate of 4.5%? (Given $(1.045)^{-12} = 0.58966386$)
4. A bond with face value of ₹ 5,000 matures at par in 12 years. The nominal rate of interest on bond is 12% per annum paid annually. What should be the price of the bond so as to yield an effective rate of return equal to 10% per annum? (Given $(1.1)^{-12} = 0.31863082$)
5. A bond has a face value of ₹1,000 and matures at par in 15 years. The nominal interest rate is 9%. What is the purchase price of the bond to yield an effective interest rate of 8%? (Given $(1.08)^{-15} = 0.31524170$)
6. Find the purchase price of a ₹ 500, 4% bond dividends payable semi-annually, redeemable at 104 in 5 years, if the yield rate is to be 6% compounded semi-annually. (Given $(1.06)^{-5} = 0.74725817$)
7. Find the purchase price of ₹ 1,000 bond, redeemable at the end of 20 years at 105, and paying annual dividends at 5%, if the yield rate is to be 8% effective. (Given $(1.08)^{-20} = 0.21454821$)
8. Find the purchase price of a ₹ 25,000, 5% bond dividends payable semi-annually, redeemable at par in 5 years, if the yield rate is to be 6% compounded semi-annually (Given $(1.06)^{-10} = 0.55839478$).
9. An investor is considering purchasing a new issue of 5-year bonds of ₹ 100,000 par value and an annual fixed coupon rate of 12% while coupon payments are made semi-annually.

The minimum yield that the investor would accept is 6.75%. Find the fair value of the bond. (Given $(1.0675)^{-10} = 0.52038068$)

10. A bond has a face amount of ₹ 1,000 and matures in 12 years. The nominal interest is 9%. What is the price of the bond that will yield an effective interest of 8%?
11. Find the purchase price of a ₹ 1,000 bond, redeemable at the end of 12 years at 15% premium and paying annual interest at 8%, if the yield rate is to be 10% per annum effective.
12. A bond has a face value of 1,000 and maturity period of 15 years. The nominal interest rate is 10%. What should be the price of the bond to yield an effective interest rate of 8%?
13. A bond with a face value of 5,000 matures at par in 12 years. The nominal rate of interest on bond is 9% p.a paid annually. What should be the price of the bond so as to yield an effective rate of return equal to 11% per annum?
14. A coupon bond has a ₹ 1,000 face value and provides a 10.5% semi-annual coupon for 14 years. The appropriate effective (or discount) rate is 9% per annum. What is the value of the coupon bond?
15. A zero coupon bond has a ₹ 1,000 face value with a 25 year life. The appropriate discount (or effective) rate is 9% p.a. What is the value of the zero coupon bond? Find out what should be the value of this bond after 8 years?

ANSWERS

1. ₹ 94,842.02	2. ₹ 8,658	3. ₹ 1,136.78	4. ₹ 5,681.40	5. ₹ 1,085.59
6. ₹ 472.18	7. ₹ 716.13	8. ₹ 23,931.50	9. ₹ 94,671	10. ₹ 1,075.36
11. ₹ 911.53	12. ₹ 1,171.19	13. ₹ 4,350.76	14. ₹ 1,118.07	15. ₹ 115.97, ₹ 231.05